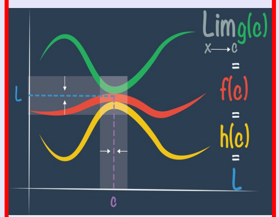
Calculus I

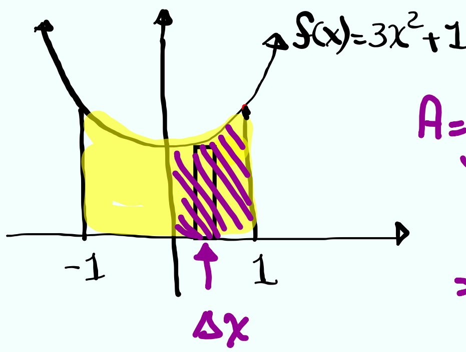
Lecture 46



Feb 19-8:47 AM

Class Quiz 17

Find the shaded area below.



$$A=\int_{-1}^{1}\left(f(x)-0\right)dx$$

$$=2\int_{0}^{1}\left[3x^2+1\right]dx$$

$$=2\left(x^3+x\right)\Big|_0^1$$

$$=2\left[1^3+1-0\right]=2\cdot 2=\boxed{4}$$

Nov 21-7:15 AM

More on integration

1) $\int f(x)\,dx = F(x) + C$ where $\frac{d}{dx}[F(x)+C] = f(x)$

2) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$. Power rule

3) $\int \cos x\,dx = \sin x + C$

4) $\int \sin x\,dx = -\cos x + C$

5) $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

6) $\int c\,f(x)\,dx = c\int f(x)\,dx$

7) $\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$ Definite Integral

8) $\int \sec x \tan x\,dx = \sec x + C$

9) $\int \csc x \cot x\,dx = -\csc x + C$

Nov 19-7:27 AM

Evaluate $\int_0^1 (\sec x \tan x - 2x + 4)\,dx$

$$= \left(\sec x - x^2 + 4x\right)\Big|_0^1$$

$$= \left[\sec 1 - 1^2 + 4(1)\right] - \left[\cancel{\sec 0}^{\,1} - 0^2 + 4(0)\right]$$

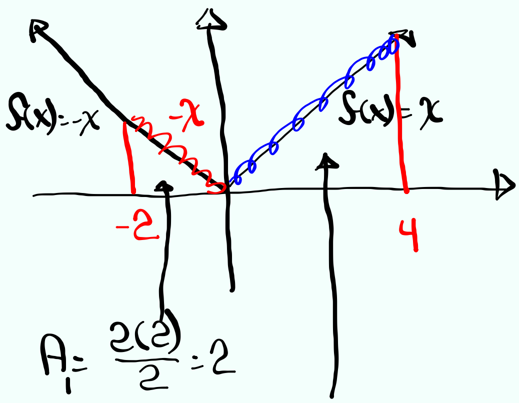
$$= \sec 1 - 1 + 4 - 1 = \sec 1 + 2$$

$$= \boxed{2 + \sec 1}$$

Nov 21-7:43 AM

Evaluate $\int_{-2}^{4} |x|\,dx = \int_{-2}^{0} -x\,dx + \int_{0}^{4} x\,dx$

$f(x) = |x|$



$A_1 = \frac{2(2)}{2} = 2$

$A_2 = \frac{4 \cdot 4}{2} = 8$

$$= -\frac{x^2}{2}\Big|_{-2}^{0} + \frac{x^2}{2}\Big|_{0}^{4}$$

$$= -\frac{1}{2}\left[0^2 - (-2)^2\right] + \frac{1}{2}\left[4^2 - 0^2\right]$$

$$= -\frac{1}{2}\cdot(-4) + \frac{1}{2}\cdot 16$$

$$= 2 + 8 = \boxed{10}$$

Nov 21-7:47 AM

Subs. Method

$$\int 2x(x^2+1)^5\,dx$$

$u = x^2 + 1$

$du = 2x\,dx$

Is there anything in the integrand such that its derivative is also part of the integrand?

$$\int 2x(x^2+1)^5\,dx = \int u^5\,du = \frac{u^6}{6} + C$$

$$= \boxed{\frac{1}{6}(x^2+1)^6 + C}$$

Nov 21-7:53 AM

find $\int x^2\sqrt{\underbrace{x^3+1}}\,dx = \int \sqrt{u}\,\frac{du}{3}$

$u = x^3+1$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$= \frac{1}{3}\int u^{1/2}\,du$

$= \frac{1}{3}\cdot\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$

$= \frac{1}{3}\cdot\frac{u^{3/2}}{\frac{3}{2}} + C = \frac{2}{9}u\sqrt{u} + C$

$= \frac{2}{9}(x^3+1)\sqrt{x^3+1} + C$

Nov 21-7:58 AM

find $\int x^3\cos x^4\,dx = \int \cos u\,\frac{du}{4}$

$u = x^4$

$du = 4x^3\,dx$

$\frac{du}{4} = x^3\,dx$

$= \frac{1}{4}\int \cos u\,du$

$= \frac{1}{4}\sin u + C$

$= \boxed{\frac{1}{4}\sin x^4 + C}$

$\frac{d}{dx}\left[\frac{1}{4}\sin x^4 + C\right] =$

$\frac{1}{\cancel{4}}\cdot\cos x^4\cdot\cancel{4}x^3 + 0 = x^3\cos x^4$

Nov 21-8:04 AM

Find $\int \cos x \sec^2(\sin x)\,dx$

$u = \sin x$

$du = \cos x\,dx$

$\int \sec^2 u\,du = \tan u + C$

$= \tan(\sin x) + C$

Nov 21-8:10 AM

$\int \frac{2x+1}{(x^2+x+8)^3}\,dx = \int \frac{du}{u^3}$

$u = x^2+x+8$

$du = (2x+1)\,dx$

$= \int u^{-3}\,du$

$= \frac{u^{-3+1}}{-3+1} + C$

$= \frac{u^{-2}}{-2} + C$

$= \frac{-1}{2} \cdot \frac{1}{u^2} + C$

$= \boxed{\frac{-1}{2(x^2+x+8)^2} + C}$

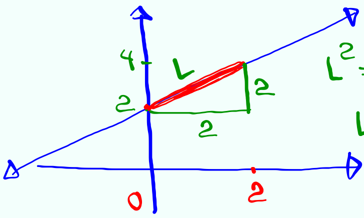
Nov 21-8:17 AM

If $f(x)$ is diff. on (a,b), then the arc length of $f(x)$ on $[a,b]$ is

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

$f'(x) = 1$

$f(x) = x + 2$



$L^2 = 2^2 + 2^2$

$L^2 = 8$

$L = \sqrt{8} = 2\sqrt{2}$

$$L = \int_0^2 \sqrt{1 + (1)^2}\, dx$$

$$= \int_0^2 \sqrt{2}\, dx$$

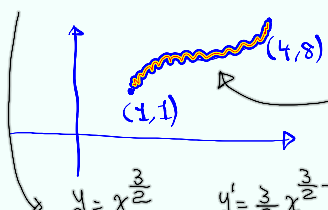
$$= \sqrt{2}\, x \Big|_0^2$$

$$= \sqrt{2}\,(2 - 0)$$

$$= \boxed{2\sqrt{2}}$$

Nov 21-8:22 AM

Find the length of the arc given by $y = x\sqrt{x}$ from $x = 1$ to $x = 4$.



$$L = \int_1^4 \sqrt{1 + [f'(x)]^2}\, dx$$

$y = x^{\frac{3}{2}}$

$y' = \frac{3}{2} x^{\frac{3}{2} - 1}$

$y' = \frac{3}{2} x^{1/2}$

$y' = \frac{3}{2}\sqrt{x}$

$1 + (y')^2 = 1 + \left(\frac{3\sqrt{x}}{2}\right)^2$

$= 1 + \frac{9}{4}x$

$$= \int_1^4 \sqrt{1 + \tfrac{9}{4}x}\, dx$$

$u = 1 + \frac{9}{4}x$ $\quad x = 1,\ u = \frac{13}{4}$ $\quad x = 4,\ u = 10$

$du = \frac{9}{4}\, dx$

$\frac{4}{9}\, du = dx$

$$\int_{\frac{13}{4}}^{10} \sqrt{u} \cdot \frac{4}{9}\, du$$

$\vdots$

Try it on your own and verify this answer

$$\frac{1}{27}\left[80\sqrt{10} - 13\sqrt{13}\right]$$

Nov 21-8:29 AM